

ABSTRACTS OF PAPERS DEPOSITED AT VINITI*

HEAT AND MASS TRANSFER IN CONVECTIVE DRYING OF A BUILDING MATERIAL IN AN AIR JET

V. K. Savin, M. Korger,
and A. M. Miksher

UDC 66.015.23:532.525

A theoretical solution for the heat-transfer coefficient has been used in an experimental comparison for a variety of circumstances; nozzles of large round and slot shapes are employed.

The observed and calculated mass-transfer coefficients have been compared on results on the sublimation of naphthalene. The comparison was made at the critical point ($r = 0$) and along a flat surface, the relevant parameter being the dimensionless distance ($\bar{h} = h/d_0$) for various nozzle diameters d_0 and jet speeds U_0 . The effects of nozzle distance from the layer were also examined for an air speed U_0 of 30 m/sec and a slot width h_0 of 5 mm. The results provide the basis for formulas for the local mass-transfer coefficients.

The suitability of average mass-transfer coefficients was examined via experimental values for \bar{Nu}_m in the drying of sand; the following were incorporated:

- 1) the effects of the flow speed on \bar{Nu}_m for $d_0 = 40$ mm, $h_0 = 150$ mm;
- 2) the effects of the distance from the nozzle to the surface of the material on \bar{Nu}_m for $d_0 = 40$ mm, $U_0 = 10$ m/sec;
- 3) the effects of nozzle diameter on Nu_m for $h_0 = 150$ mm, $U_0 = 21.5$ m/sec.

The observed and calculated values were in good agreement.

These studies show that it is possible to use the equations for the mean and local mass-transfer coefficients in calculations.

Dep. 2290-74, February 11, 1974.

Original article submitted January 1, 1973.

HEAT- AND MASS-TRANSFER CHARACTERISTICS OF GRANULATED SUGAR

M. A. Volkov, M. A. Gromov,
and V. D. Mikhailov

UDC 536.2.02

Measurements have been made of the thermal conductivity, thermal diffusivity, and specific heat for granulated sugar in relation to density. The following empirical formulas can be used to approximate the results:

*All-Union Institute of Scientific and Technical Information.

Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 28, No. 2, pp. 346-373, February, 1975.

©1976 Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for \$15.00.

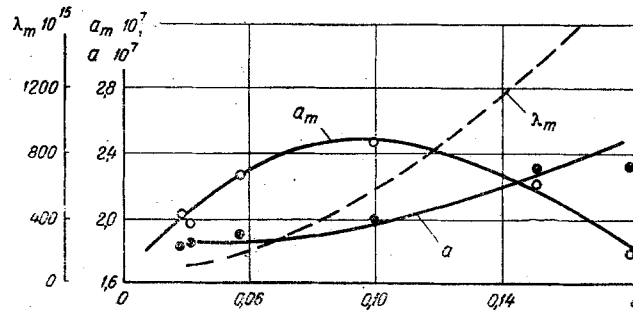


Fig. 1. Effects of water content on diffusion coefficient, mass transmission factor, and thermal diffusivity for granulated sugar.

$$a = (2.28 \cdot 10^{-3} \gamma - 0.6) \cdot 10^{-7};$$

$$c = 1170 + 0.126 (\gamma - 800).$$

The relation of thermal conductivity λ to density γ is

$$\lambda = (2.28 \cdot 10^{-3} \gamma - 0.6) c \gamma \cdot 10^{-7}.$$

The conductometric method of measuring granulated-sugar water content [1] enables one to record directly the change in water content at any point in the specimen without sampling, and this enables one to examine the linked heat and mass transfer. The results indicate that the heat- and mass-transfer coefficients are dependent on the water content (Fig. 1).

It is found that the Lykov number is close to 1.0 for this range of water contents (0.04-0.18%); this confirms that the heat and mass transfer are coupled in this material.

The equilibrium water content and thermodynamic parameters of granulated sugar [1, 2] provide a systematic classification of the major heat- and mass-transfer characteristics: the true isothermal specific heat, the thermal-gradient coefficient, and the water diffusion coefficient for water contents from 0.028 to 0.18% for ash contents from 0.020 to 0.068% at 280-300°K.

NOTATION

- ρ is the density, kg/m³;
- c is the specific heat, J/kg-K;
- λ is the thermal conductivity, W/m-K;
- a is the thermal diffusivity, m²/sec;
- a_m is the diffusion coefficient for water, m²/sec;
- u is the water content, kg/kg;
- Lu is the Lykov number;
- λ_m is the mass-transmission factor, kg-mol/m · J · sec.

LITERATURE CITED

1. M. A. Volkov and V. D. Mikhailov, "Mass-transfer coefficients for granulated sugar," Sakh. Prom-st., No. 11, 29 (1973).
2. M. A. Volkov, O. B. Tserevitinov, and V. D. Mikhailov, "Equilibrium water content and thermodynamic parameters of granulated sugar," Sakh. Prom-st., No. 6, 43 (1973).

Dep. 2277-74, March 25, 1974.

Original article submitted October 5, 1973.

HEAT-UTILIZATION EFFICIENCY IN EVAPORATION OF A LIQUID THROUGH A MEMBRANE

V. D. Volzin

UDC 536.24.02

There are the following four types of equipment using liquid evaporation through membranes: 1) evaporation into vacuum, 2) evaporation into a layer of immobile gas, 3) evaporation into a gas flow, and 4) evaporation through the pores in the membrane made of a material not wetted by the liquid. The heat utilization in such equipment is examined, i.e., the consumption of heat per unit mass of vapor. The physical transport patterns for the vapor in cases 1, 2, and 4, are very similar, and they are therefore described by similar equations. The overall heat flux is made up of the heat transported by the temperature gradient and that transported by the convective vapor flux. The second is determined by the properties of the membrane, while the first must be estimated via the known temperature distribution along the gap between the evaporation and condensation surfaces. This distribution has been determined by solving the differential equations for convective thermal conduction. The case of type 3 is considered separately, via an idealized scheme, in which the surface of the membrane is flushed by gas that is completely saturated with the vapor, after which the gas flow is cooled to another temperature, with the vapor content again attaining the equilibrium value. In all cases, analytical expressions are derived for the heat-utilization efficiency on the basis of data on the throughput and evaporation and condensation temperatures.

A numerical calculation has been performed for a throughput of 10 liters/m²·h of water with evaporation and condensation temperatures of 50 and 0°C, respectively; the following efficiencies have been found: 1) 97%; 2) 83; 3) 82.5; and 4) 81.5.

Conclusions are drawn on the choice of gas and temperature difference for the evaporation and condensation surfaces.

Dep. 2296-74, April 19, 1974.

Original article submitted February 14, 1973.

AN ELECTRICAL-CONTACT METHOD OF EXAMINING THE HYDRODYNAMIC STRUCTURE OF A THREE-PHASE IMMOBILE BED

V. A. Kirillov, M. A. Kasamanyan,
and V. A. Kuzin

UDC 532.546

The electrical contact method is now widely used to examine two-phase flows. Here it has been applied to the hydrodynamic structure of a three-phase immobile bed, with liquid and gas passing upwards through it.

Methods. The apparatus consisted of a column having $D = 100$ mm and height 1200 mm, which was filled with glass spheres having $d = 18$ mm, and a transducer connected to a measuring system incorporating an IRA-5 computer. The transducer was a sphere having $d = 18$ mm, and this at five points bore cylinders, which were fitted with stainless steel surgical needles having $d = 0.5$ mm with the ends projecting 6.5 mm. The cylinders served as one electrode and the needles as the other. The transducers were supplied with ac at 200 kHz. The operation was based on the following effect: there is a marked change in the resistance between the electrodes when a gas bubble passes, which is recorded as a voltage pulse at the IRA-5. The apparatus allows for recording the width and amplitude of the pulse, the latter corresponding to the distance from the side.

Results. The data were processed via the theory of random processes to give the mathematical expectation, dispersion, autocorrelation functions, spectral density, and rms frequencies in the spatial and time coordinates.

It was found that there are three hydrodynamic states: 1) the bubble state with a gas content $\varphi = 0.2$; 2) formation of highly conducting channels ($\varphi = 0.2-0.6$); and 3) dispersed annular flow ($\varphi > 0.6$).

It was found that the liquid phase may be considered as a film oscillating with the frequency of bubble passage, while the gas can be treated as a continuous medium.

Dep. 2269-74, May 7, 1974.

Original article submitted November 24, 1973.

EXPERIMENTAL STUDY OF THE METHOD OF
LIGHT SCATTERING FOR THE CONTINUOUS
MEASUREMENT OF THE CONCENTRATION OF
THE DISPERSE PHASE IN AERODISPERSED SYSTEMS

A. P. Klimenko and V. I. Korolev

UDC 532.529.5

An evaluation of the possibility of using the method of integral light scattering to determine the concentration of particles of different types of industrial dust, which are, in general, polydisperse, in a stream is of considerable interest. For this purpose an experimental study of the method was conducted on the following types of dust: coal (fine fraction), quartz-containing, and flour. The coal and quartz-containing dust belong to the monodisperse systems with a mean particle size of 2μ (for coal dust) and 4μ (for quartz-containing dust). In contrast to them the flour dust is polydisperse with a wide range of variation in particle size. With some approximation it can be represented in the form of a mixture of three monodisperse systems with mean particle sizes of 5, 21, and 34μ , with their relative content in the mixture equal to 14:4:1, respectively, i. e., the particles with a size of 5μ predominate.

Thus, all the types of dust studied pertain to large particles whose size considerably exceeds the wavelength λ of the illuminating light. For such particles the intensity of the scattered light is proportional to the cross section of the particle and does not depend on the wavelength λ . A study of the scattering indicatrices of coal, flour, and quartz-containing dust showed that setting the measuring photoreceiver at a 130° angle to the optical axis of the illuminating light flux is the most efficient.

The tests were conducted on an operating dust chamber. The dust concentration at the site where the detector was mounted was monitored by the gravimetric method. The studies conducted showed that in the range of variation in the dust concentration c from 10 to 200 mg/m^3 there is a linear dependence between c and the output signal u of the measuring system. The linear dependence of u on c is disrupted at concentrations exceeding 200 mg/m^3 , which is connected with the fact that multiple scattering begins to have an effect at such high concentrations.

A comparative estimate of the results obtained for different types of dust shows that the concentration sensitivities for quartz-containing and flour dust coincide and equal $14 \text{ mV} \cdot \text{mg}^{-1} \cdot \text{m}^3$, while for coal dust the sensitivity is 2.3 times lower. These differences can be explained by the fact that coal dust represents fully reflecting particles with smooth surfaces, while flour and quartz-containing dust represent "white" particles with complete but diffuse scattering.

The results of the measurements were analyzed statistically to determine the degree of probability of the connection between the values of the dust concentration obtained by the weight method and by the light-scattering method. The calculation showed that for the linear section of the curves the correlation coefficients are 0.84 for coal dust, 0.93 for quartz-containing dust, and 0.96 for flour dust, i. e., a rather close correlation connection exists which varies within small limits depending on the type of dust.

An estimate of the accuracy of the measurements showed that for the linear section of the curve $u = f(c)$ the reduced error is $\pm 10\%$.

Dep. 2301-74, April 12, 1974.

Original article submitted May 21, 1973.

DETERMINATION OF PHASE CONTACT TIME IN DEVICES CONTAINING NOZZLES

N. V. Ostapchuk, T. N. Grosul,
G. S. Zelinskii, L. D. Kamyshnik,
and V. I. Aleinikov

UDC 532.546

A method is presented for calculating the time that particles stay in the disperse phase in heat- and mass-exchange devices built in the form of a vertical stack containing series of nozzles mounted horizontally. The vertical particle motion produced by gravitational forces is characterized by the presence of local streams formed as a result of the interaction of the particles with the nozzles and by an oppositely directed stream of the dispersion medium. The average velocity of the disperse stream in this case is determined by the geometry, the structural properties of the device, and the parameters of the dispersion medium. The arbitrary division of the height of the device into individual sections i of height h by cross sections passing through the series of nozzles allows one to calculate the average particle velocities at the start and end of each section.

The average velocity v_{i+1}^S of the disperse phase at the start of the $(i + 1)$ -th section is determined from the calculated average velocity v_i^e at the end of the i -th section on the basis of the equality of the number of particles, assuming them to be absolutely elastic spheres, before and after their encounter with a nozzle. When the particles of the disperse phase are considered as absolutely inelastic spheres their velocity after an encounter with a nozzle is taken as zero.

The relationship $v_{i+1}^S = cv_i^e$ is determined by the coefficient c , which allows for the size and shape of the cross section of a nozzle, the number of them in a series, the horizontal distance between nozzles, and the vertical distance between series of nozzles. Expressions for the coefficient c for nozzles having cross sections of round, square, and hexagonal shape and with various nozzle orientations were obtained using the geometrical probability of the encounter of a particle with a nozzle of the series under consideration. From the known average velocity of the disperse phase at the start of each section and the velocity of swirl of its particles it becomes possible to calculate (by equations proposed by Z. R. Gorbis, 1964) the time τ_i for the passage of the particles through the sections.

The time τ that the disperse phase stays in a heat- and mass-exchange device is determined by the sequential calculation of the τ_i on a computer and their subsequent summation. A comparison of the calculated time ($\tau_c = 2.87$ sec) and the actual time ($\tau_a = 3$ sec) of interaction of a grain of wheat and the drying agent in the heating chamber of a Tselinnyi-50 recirculation grain drier showed an identity of results and the applicability of the calculating method of determining τ . The use of the method developed has considerable importance for the study, calculation, design, and optimization of heat- and mass-exchange devices of this type.

Dep. 2302-74, June 5, 1974.

Original article submitted October 9, 1973.

CALCULATION OF THE FILTRATION DISCHARGE OF FREE-FLOWING MATERIALS FROM HOPPERS

V. E. Davidson, V. I. Eliseev,
and A. P. Tolstopyat

UDC 621.547

A model is constructed for the filtration discharge of a free-flowing material based on the filtration characteristics of the charge-bunker system [1], a criterial equation for gravitational discharge [2], and the assumption that the gas filtration does not disturb the dynamic arch at the discharge opening but is manifested in the application of an additional force load on the particles passing through this arch.

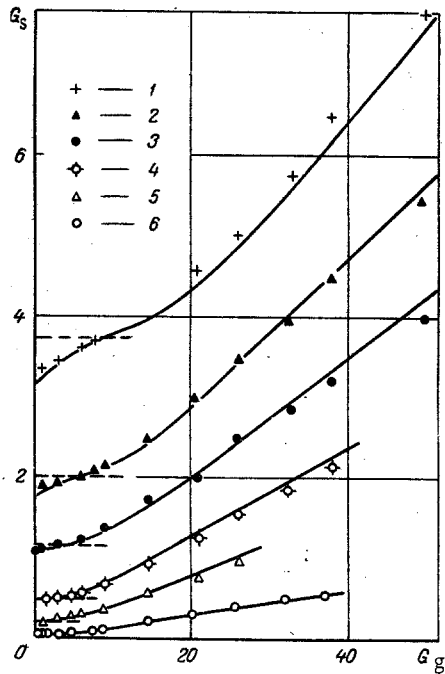


Fig. 1

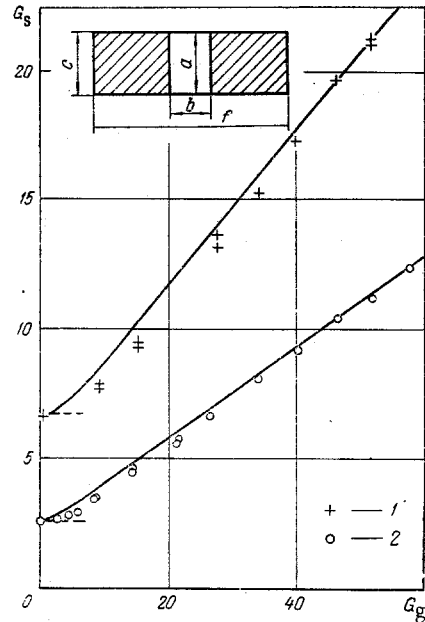


Fig. 2

Fig. 1. Comparison of experimental data of [3] with calculation (solid lines). Polystyrene: $d=3.2$ mm; $\gamma_S = 1.05$ g/cm³ (G_S , kg/sec; G_g , g/sec); 1) $\bar{f}=3.42$; 2) 4.88; 3) 6.84; 4) 11.4; 5) 17.1; 6) 34.2. G_{gr} : dashed line. $G_S = G_{gr}$ when $a_f=0$; $Re=0-340$.

Fig. 2. Comparison of experimental data with calculation (solid lines). Lead: $d=2.75$ mm; $\gamma_S = 11.3$ g/cm³ (G_S , kg/sec; G_g , g/sec): 1) $f=2.4$; 2) 4.0. G_{gr} : dashed line. $G_S = G_{gr}$ when $a_f=0$; $Re=0-1970$. Above: diagram of bottom of flat hopper.

A new value a_f is included in the criterion of the reduced flow rate, and the equation for the flow rate of a free-flowing material discharging from the opening in the bottom of a hopper with allowance for filtration is written in the form

$$G_s = K(\delta) \gamma_g D^{2.5} (g \pm a_f)^{0.5} = G_{gr} \left(1 \pm \frac{a_f}{g} \right)^{0.5}. \quad (1)$$

The "plus" and "minus" signs are determined by the filtration of the gas accompanying the discharge or opposite to it.

The value of a_f , which is determined by the filtration forces acting on a particle, was found on the assumption that the gas discharge is isochoric and the process of filtration discharge of the free-flowing material from the hopper is steady. This made it possible to separate out the flow rate of the gas which is moving relative to the particles and is a source of additional forces:

$$G_{gf} = G_g - \frac{\gamma_g}{\gamma_g} \cdot \frac{1 + \varepsilon}{1 - \varepsilon} G_s. \quad (2)$$

The value of a_f was determined from the forces found [1] (with allowance for the presence of separation zones) and the mass of a particle, as a result of which the equation for determining the filtration flow rate of a free-flowing material can be written in the form

$$G_s = G_{gr} \left\{ 1 \pm \frac{\Delta p}{H \gamma_c} \cdot \frac{\xi_f}{\xi} [(1 - \varepsilon) k_d \psi_2 + E \psi_1] \right\}^{0.5}. \quad (3)$$

Let us write one other form of Eq. (3):

$$G_s = G_{gr} \left\{ 1 \pm \frac{27}{4} \xi_f Re^2 \frac{\mu^2 g}{\gamma_s d^2 \gamma_g} \left(\frac{1 - \varepsilon}{\varepsilon} \right)^2 \left[\frac{1 - \varepsilon}{\varepsilon} k_d \psi_2 + E \psi_1 \right] \right\}^{0.5}. \quad (4)$$

Here

$$\text{Re} = \frac{2}{3} \cdot \frac{Ggfd}{(1-\varepsilon)F\mu g}; \quad k_d = [1 + T(\bar{f}-1)] \left[\frac{1}{\bar{f}} + 7 \cdot 10^{-5} \cdot \text{Re}(\bar{f}-1) \right];$$
$$\psi_1 = \frac{\varphi_0}{4} - \frac{1}{8} \sin 2\varphi_0; \quad \psi_2 = \frac{1}{2} (1 - \cos^3 \varphi_0) - \frac{3}{4} \sin^2 \varphi_0 \cos \varphi_0;$$
$$E = (50/\text{Re} + 0.446\bar{f}) \bar{f}/\xi_f.$$

The values of ξ , ξ_f , and T are determined from equations obtained in [1]. The angle of stream separation $\varphi_0 = 110^\circ$ for fully turbulent flow was used in the calculations.

A comparison of the results of calculation by Eq. (4) with the experimental data of [3] obtained on a flat hopper (the free-flowing material was polystyrene) and with additional experiments with lead shot is presented in Figs. 1 and 2. The experimental data of [4] obtained on round hoppers were checked against Eq. (4). In all the calculations performed the disagreement between the experimental points and the results obtained from Eq. (4) did not exceed 10%.

A comparison with the data of [3] also showed that the ratio a_f/g reached 500 and that in this case the flow rate of the free-flowing material varied in accordance with Eq. (1).

Thus, the filtration of gas allows one to pass as much free-flowing material as desired through openings of limited size, as well as to regulate it smoothly.

NOTATION

| | |
|-------------------|---|
| G | is the weight flow rate; |
| γ | is the specific weight; |
| ε | is the porosity; |
| D | is the diameter of discharge opening; |
| g | is the acceleration of gravity; |
| H | is the height of layer of free-flowing material; |
| Δp | is the excess pressure in hopper in comparison with the medium into which the discharge occurs; |
| F | is the cross-sectional area of hopper; |
| d | is the equivalent diameter of particles of free-flowing material; |
| $\underline{\mu}$ | is the viscosity of gas; |
| f | is the ratio of cross-sectional area of hopper to area of discharge opening. |

Subscripts

| | |
|----|---------------------|
| s | is the solid phase; |
| g | is the gas; |
| f | is the filtration; |
| gr | is the gravitation; |
| c | is the charge. |

LITERATURE CITED

1. V. E. Davidson, V. I. Eliseev, and A. P. Tolstopyat, *Inzh.-Fiz. Zh.*, 28, No. 1 (1975).
2. F. E. Keneman, *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekhan. i Mashinostr.*, No. 2 (1960).
3. V. E. Davidson and A. P. Tolstopyat, *Inzh.-Fiz. Zh.*, 23, No. 4 (1972).
4. A. G. Tsubanov, *Inzh.-Fiz. Zh.*, 17, No. 2 (1969).

Dep. 2312-74, April 19, 1974.

Dnepropetrovsk State University.

Original article submitted May 22, 1972.

STUDY OF FOUNTAIN EFFECT MODES IN ROUGH CONICAL DEVICES

A. V. Golubkovich

UDC 532.546

A number of fountain effect modes are obtained for counterflow of the gaseous and solid phases in rough conical devices of the gap type. The hydrodynamics of fountain effect processes was studied both in diluted and in dense phases (with flow of a jet through a suspended layer of material).

The following types of fountain effect are characteristic of the diluted phase: vortex, pulsed, and transitional; for the dense phase the following fountain effect modes are characteristic: membrane, free, and compartmented.

A detailed description of the hydrodynamics of the fountain effect processes for each phase is given in the article and it is shown how the roughness of the walls of the apparatus affects the range of the individual fountain effect modes compared with smooth devices.

The critical velocities and the total pressure losses in each phase, which are obtained through a criterial analysis of the experimental results, are presented for each mode.

Dep. 2305-74, April 17, 1974.

Original article submitted June 14, 1973.

STUDY OF FLOW THROUGH NOZZLES OF BOILING LIQUID PROPANE WITH PHASE TRANSITIONS

S. V. Rubinshtein

UDC 622.691.53.002(211)

Multiple disseminations of an elastic medium develop as a result of evaporation during the flow of a saturated liquid. Consequently, the appearance of crisis effects is possible during the flow of this two-phase stream.

As a result of the solution of the system of equations of the gasdynamics of two-phase streams [1] together with equations characterizing the rate of phase transitions in streams of boiling liquid [2] and the equation for the velocity of sound propagation in a two-phase medium [3] one can show that the crisis effects during the flow are functions of the hydrodynamic homochrony number Ho and of the dimensionless complex K which characterizes the ratio of the energy of the system to the energy of formation of the nucleus of the new phase.

In order to obtain the dependence of the critical pressure ratio on these similarity criteria experimental studies were made of the process of flow of boiling liquid propane and a propane-butane mixture by the method presented in [2]. The studies were conducted by setting up a series of experiments, each of which corresponded to a certain value of the dimensionless complex K ranging from $1.1 \cdot 10^3$ to $4.5 \cdot 10^5$ and to values of the hydrodynamic homochrony of from 2 to 30.

Through an analysis of the experimental data by rules of mathematical statistics an equation of the following type was obtained:

$$\beta_{cr} = (0.57 + k \cdot 10^{-7}) (1 - e^{-0.5Ho}).$$

The experimental studies of the process of flow of boiling water through cylindrical nozzles performed by Polyakov [4] also agree with the equation (1) obtained, which widens the range of its applicability.

The equation obtained enables one to develop a method of hydraulic calculation of through cross sections, as well as the parameters of a stream of boiling liquid with different modes of flow: subcritical, transitional, and critical.

LITERATURE CITED

1. M. E. Deich and G. A. Filippov, Hydrodynamics of Two-Phase Media [in Russian], Energiya, Moscow (1968).
2. S. V. Rubinshtein, V. A. Ivanov, and A. K. Karalyuk, Inzh.-Fiz. Zh., 23, No. 1 (1972).
3. S. V. Rubinshtein and Yu. S. Pogorelov, The Use of Gas in the National Economy. Proceedings of the Giproniigaz Institute [in Russian], No. 9, Saratov (1971).
4. K. S. Polyakov, Proceedings of the Leningrad Polytechnic Institute, Turbines [in Russian], No. 247, Mashinostroenie (1965).

Dep. 2303-74, July 8, 1974.

Original article submitted March 16, 1973.

RESISTANCE OF CYCLONE CHAMBERS IN THE
NONSELF-SIMILAR REGION OF STREAM FLOW

É. N. Saburov and S. V. Karpov

UDC 532.54.001.5

The study was performed on a cyclone chamber with a working volume 201 mm in diameter. Its relative geometrical characteristics were varied in the following ranges: $L = 0.5-1.57$; $f_{in} \cdot 10^2 = 3.402-17.009$; $d_{out} = 0.2-0.6$; $\Delta \cdot 10^2 = 0-0.995$.

Some of the experimental data are presented in Fig. 1 as an example. The resistance of cyclone-vortex chambers and the losses in them are mainly determined by the rotational motion of the stream. The fraction of the losses to the friction of the stream against the walls of the working volume (and of the output and input channels) is relatively small in the total loss for smooth-walled chambers and increases with an increase in the relative roughness. However, even in smooth-walled chambers the friction of the stream against the wall (with relatively small losses directly to friction) has an effect on the level of the rotational velocities in the working volume and, consequently, on the losses in reaching a predetermined level of twist and the predetermined input and output components of the total coefficient of resistance. The dependence of ζ on Re_{in} is opposite to the variation of the coefficient of frictional resistance.* The experiments showed that the overall nature of the variation in ζ as a function of Re_{in} is connected with the ratio of the thickness of the laminar sublayer at the wall of the working volume to the heights of the roughness projections. The resistance of a cyclone chamber does not depend on Re_{in} (the flow becomes self-similar) only in the case when the roughness protuberances pass considerably beyond the limits of the laminar sublayer; the retarding effect of the walls will be determined by the resistance of the shape of the protuberances — by the resistance of the bluff roughness projections. In general, the variation in ζ with an increase in Re_{in} can be represented as follows: in a laminar mode ζ will probably increase and reach a maximum at the critical Reynolds number, and in the transitional mode ζ decreases; in the turbulent intermediate mode, in contrast to the two previous modes, the nature of the variation in ζ varies considerably with Δ and either an increase or a decrease in ζ can occur depending on Δ ; in the mode of developed roughness the total coefficient of resistance does not vary. The smaller the Δ , the larger the range of Re_{in} in which the nature of the variation in ζ for rough chambers corresponds to a smooth chamber. The Reynolds number Re_{in}^{sel} starting with which ζ is almost independent of Re_{in} (the boundary of the second self-similar region) can be determined with an accuracy of 5% with respect to ζ by the equation

*The mechanism of the similarity effect was first explained by I. E. Idel'chik.

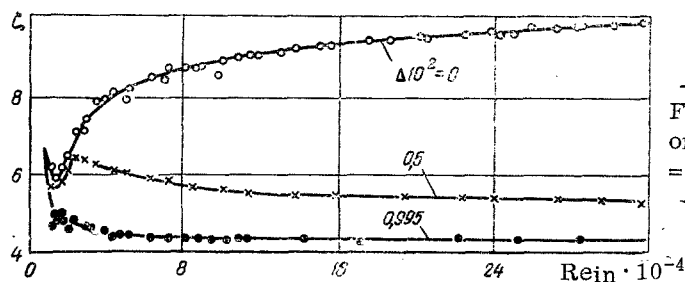


Fig. 1. Dependence of total coefficient of resistance of chamber on Re_{in} ($d_{out} = 0.4$; $f_{in} = 10.205 \cdot 10^{-2}$; $L = 1.57$).

$$\text{Re}_{\text{in}}^{\text{sel}} \cdot 10^{-4} = \exp [6 (\Delta \cdot 10^2 - 1.333) (f_{\text{in}} - 0.355) + 0.95] - 7.46 (\ln d_{\text{out}} + 0.916) (\Delta \cdot 10^2 - 1.12). \quad (1)$$

The Reynolds number corresponding to the end of the transitional mode of flow and to the start of the turbulent mode is

$$\text{Re}_{\text{in}}^{\text{t.in}} \cdot 10^{-4} = 0.66 - 4.7 f_{\text{in}} + d_{\text{out}} + 0.153L. \quad (2)$$

For smooth-walled chambers

$$\xi = 145.6 f_{\text{in}}^{1.531} d_{\text{out}}^{-1.646} \exp (-1.35L) \text{Re}_{\text{in}}^n$$

where

$$n = 0.116 - 0.684 f_{\text{in}} + 0.069L \text{ when } d_{\text{out}} = 0.2$$

and

$$n = 0.143 \exp (0.535L - 5.925 f_{\text{in}} - 1.195 d_{\text{out}}) \text{ when } d_{\text{out}} = 0.3 - 0.6. \quad (3)$$

For rough cyclone-vortex chambers in the range of Reynolds numbers which is most suitable and most often encountered in practice the mode of flow of the stream can be assumed to be self-similar.

NOTATION

| | | |
|---|---|---|
| \bar{D}, \bar{L} | are the diameter and length of working volume of cyclone chamber; | |
| \bar{V}_{in} | is the average stream velocity in slits; | |
| \bar{P}_t | is the total pressure drop in chamber; | |
| $\bar{\rho}, \bar{\nu}$ | are the density and kinematic viscosity of stream; | |
| $\text{Re}_{\text{in}} = \bar{V}_{\text{in}} \bar{D} / \bar{\nu}$ | is the input Reynolds number; | |
| $\xi = 2\bar{P}_t / \bar{\rho} \bar{V}_{\text{in}}^2$ | is the total coefficient of resistance of chamber. | |
| $L = \bar{L} / \bar{D}$ | is the length; | } relative geometric characteristics of chamber |
| $d_{\text{out}} = \bar{d}_{\text{out}} / \bar{D}$ | is the diameter of output opening; | |
| $f_{\text{in}} = 4\bar{f}_{\text{in}} / \pi \bar{D}^2$ | is the area of entrance; | |
| $\Delta = \bar{\Delta} / \bar{D}$ | is the roughness of working volume. | |

Dep. 2284-74, June 19, 1974.

Original article submitted June 6, 1973.

NEW RHEOLOGICAL EQUATIONS FOR VISCOPLASTIC MEDIA

A. Ya. Malkin, I. A. Glushkov,
and V. A. Rozhkov

UDC 532.135

A special feature of the rheological properties of a large number of diverse systems (plastic dispersions and pastes, filled polymer compositions, etc.) is an abrupt change in the shear velocity by several tenths of an order of magnitude in the region of the conditional yield point. This fact gives rise to the specific properties of the rheological behavior of such systems in a nonuniform stress field.

In the report we consider an approach to the construction of rheological equations which rather fully reflect the nature of the flow curves for the class of systems being discussed. One of these equations which take into account the important anomaly in viscosity in the region of shear stresses exceeding the yield point τ_0 has the form

$$\dot{\gamma} = \begin{cases} 0 & \text{when } \tau \leq \tau_0, \\ K(\tau - \tau_0)^n & \text{when } \tau > \tau_0. \end{cases} \quad (1)$$

The special features of the rheodynamics of viscoplastic media for which Eq. (1) is applicable are examined on the example of the problem of their flow in a straight tube of length L with a circular cross section of radius R .

The dependence between the stream parameters in dimensionless form when Eq. (1) is used is given by the equation

$$q = (1 - \beta\alpha)^{n+1} \left[1 + \frac{2}{n+2} (\beta\alpha) + \frac{2}{(n+1)(n+2)} (\beta\alpha)^2 \right] - (1 - \alpha)^{n+1} \left[1 + \frac{2}{n+2} \alpha + \frac{2}{(n+1)(n+2)} \alpha^2 \right] \beta^{n+3}, \quad (2)$$

where

$$q = \frac{Q}{\pi K} \cdot \frac{n+3}{R^{n+3}} \left(\frac{2L}{P} \right)^n, \quad \alpha = \frac{\tau_y}{\tau_0}, \quad \beta = \frac{\tau_y}{\tau_R}.$$

The difference in the rheodynamics of systems with an abrupt change in the deformation rate in comparison with the usually used equations which describe a monotonic nature of the dependence $\dot{\gamma}(\tau)$ is shown on the basis of an analysis of Eq. (2).

Dep. 2297-74, April 15, 1974.

V. A. Topchiev Institute of Petrochemical Synthesis,
Academy of Sciences of the USSR, Moscow.

Original article submitted June 14, 1973.

MASS EXCHANGE IN MULTICOMPONENT MIXTURES DURING TURBULENT FLOW IN A TUBE

A. V. Kuznechikov and E. N. Konstantinov

UDC 518.12:66.015.23

The boundary problem of intraphase mass exchange for turbulent flow of a multicomponent mixture in a tube is formulated and realized on an M-222 electronic computer. A system of equations of convective diffusion and experimental functions on the axial velocity distribution and the coefficient of vortex diffusion are used in this problem. The molecular diffusion equation is written in the Stefan-Maxwell form. The boundary conditions contain the concentration at the wall of the tube, the composition of the gas entering the tube, and the condition of symmetry of the concentration profile.

The boundary problem is solved by the grid method using an implicit three-layer scheme.

A comparison between the numerical results and experiment is presented.

Dep. 2284-74, May 7, 1974.

Original article submitted June 8, 1973.

DETERMINATION OF THE COEFFICIENT OF TURBULENT DIFFUSION OF DYNAMICALLY ACTIVE ADMIXTURES

S. P. Grekov and A. E. Kalyusskii

UDC 533.72

In the article we present the results of mathematical modeling of the processes of movement of an admixture in a channel in the presence of gas exchange with the wall at the boundary of the stream, allowing one to find the numerical values of the coefficients of turbulent diffusion of dynamically active admixtures.

The idea of the proposed method is based on the dependence of the coefficient of turbulent diffusion on the velocity profile of the stream, which, in turn, depends on the concentration of the dynamically active admixture.

Let c be the concentration of the admixture. The concentration distribution of a dynamically active

admixture in time and space is adequately described by the following nonlinear boundary problem:

$$\frac{\partial}{\partial x} \left[\psi(c) \frac{\partial c}{\partial x} \right] + \frac{1}{r} \cdot \frac{\partial}{\partial r} \left[r \psi(c) \frac{\partial c}{\partial r} \right] = u_0 \frac{\partial c}{\partial x} + \frac{\partial c}{\partial t}; \quad (1)$$

$$c(x, r, 0) = c_1, \quad c(\infty, r, t) = c_1, \quad c(0, r, t) = f(t),$$

$$\left. \frac{\partial c}{\partial t} + \alpha c \right|_{r=r_0} = \alpha c_2,$$

where ψ is the coefficient of turbulent diffusion, m^2/sec ; x and r are the spatial coordinates, m ; t is the time.

On the assumption that the function $\psi(c)$ has the form

$$\psi = \psi_0 n (c - c_2)^{k-1},$$

where ψ_0 is the coefficient of turbulent diffusion for passive admixtures, the solution of the boundary problem (1) is obtained in the form

$$c(x, r, t) = \left\{ \sum_{n=1}^{\infty} \left[\left(\frac{2 [(f(\tau - \delta) - c_2)^k - (c_1 - c_2)^k] J_0(\lambda_n)}{\lambda_n [J_1^2(\lambda_n) + J_0^2(\lambda_n)]} + J_0(\lambda_n) r_0 \alpha \psi_0 c_1^k \int_0^{\tau - \delta} \exp(\lambda_n^2 \theta) d\theta \right) \frac{x}{2\delta \sqrt{\pi \delta} r_0} \times \right. \right. \\ \left. \left. \times \exp \left(-\frac{u_0 r_0}{4\psi_0^2} + \lambda_n^2 \delta - \frac{x^2}{4\delta r^2} + \frac{u_0 x}{2\psi_{cp}} \right) \right] d\delta - J_0(\lambda_n) r_0 \alpha \psi_0 c_1^k \int_0^{\tau} \exp(\lambda_n^2 \delta) d\delta \right\} J_0 \left(\lambda_n \frac{r}{r_0} \right) + (c_1 - c_2)^k \left. \right\}^{\frac{1}{k}} + c_2.$$

The numerical values of these parameters which realize the minimum in the rms error are determined on the basis of a comparison of theoretical and experimental data on the start-up of carbon dioxide gas in mining through the "playback" of different values of the parameters n and k and the calculation of the theoretical concentrations on an electronic computer by the appropriate program.

The final equation for the coefficient of turbulent diffusion of carbon dioxide has the form

$$\psi = 51.25 \sqrt{\bar{\alpha}} (u_0 d)^{-0.25} c^{1.25}, \quad m^2/\text{sec}.$$

Here u_0 is the stream velocity, m/sec ; $\bar{\alpha}$ is the coefficient of aerodynamic resistance, $\text{kg} \cdot \text{sec}^2/\text{m}^4$.

By a similar method one can obtain the values of the coefficient of turbulent diffusion for other active gases, as well as for various phlegmatizers.

This permits one to perform practical calculations connected with the use of inert admixtures and phlegmatizers, in the localization and neutralization of explosions in mining, for example.

Dep. 2285-74, May 10, 1974.

Original article submitted April 19, 1973.

ERRORS IN THE EXPERIMENTAL DETERMINATION OF THE THERMOPHYSICAL CHARACTERISTICS OF CHEMICALLY REACTIVE MATERIALS

B. A. Arutyunov, V. V. Vlasov,
A. A. Gukhman, and S. V. Mishchenko

UDC 678.072:536.081.7:[53.088.22]

In this study we give an estimate of those systematic errors in the experimental determination of the thermophysical characteristics of chemically reactive materials which result from: 1) the impossibility of maintaining the required boundary conditions; 2) the introduction of a heat-flux sensor; 3) the boundedness of a semi-infinite specimen.

These factors distort the desired temperature field determined by the solution of a heat-conduction boundary-value problem describing heat transfer in a one-dimensional semi-infinite object with boundary conditions of the first kind and continuously acting internal sources of heat. The estimate is made by comparing the given temperature field with the temperature fields of corresponding boundary-value problems which take account of the above-mentioned sources of systematic experimental errors.

On the basis of the estimates obtained, we select the geometric dimensions of the cylindrical specimen, the coordinate for the placement of the heat-flux sensor, and the necessary number of sections of the electric heater used for establishing a condition of adiabaticity on the lateral surface of a given cylindrical specimen for a given precision of the experiment.

The estimates given for the systematic errors in the experimental determination of thermophysical characteristics are illustrated by a calculation, performed on the Odra 1201 digital computer, for the phenol-formaldehyde composition K-18-2.

Dep. 2274-74, April 19, 1974.

Original article submitted July 14, 1973.

TAKING ACCOUNT OF THE THERMAL CAPACITY
OF A HEAT SOURCE IN MEASUREMENTS OF
THERMAL CONDUCTIVITY BY A
NONSTATIONARY METHOD

V. A. Tsymarnyi and N. F. Potienko

UDC 536.6

We consider the solution of the problem of heat conduction for an infinitely thin and infinitely long linear heat source generating a constant thermal power in an infinite homogeneous medium. The temperature at the surface of the source depends on the thermal properties of the medium and varies with time as follows:

$$T(r_0, \tau) = \frac{q}{4\pi\lambda} \left(\ln \frac{4a\tau}{r_0^2} - \gamma + \frac{r_0^2}{4a\tau} - \dots \right). \quad (1)$$

This approximate solution is noteworthy for the fact that at large values of Fo and nearby instants of time we can eliminate from it one of the unknown thermophysical characteristics of the medium under investigation and calculate the thermal conductivity by the formula

$$T_2(r_0, \tau_2) - T_1(r_0, \tau_1) = \frac{q}{4\pi\lambda} \ln \frac{\tau_2}{\tau_1}. \quad (2)$$

To determine more precisely the region of applicability of the solution (1), we compared it with a solution which takes account of the finite dimensions of the source:

$$T(r_0, \tau) = \frac{2q\omega^2}{\pi^3\lambda} \int_0^{\infty} \frac{(1 - e^{-\text{Fo}x^2}) dx}{x^3 [xJ_0(x) - \omega J_1(x)]^2 + [xY_0(x) - \omega Y_1(x)]^2}. \quad (3)$$

An analysis of the differences between the solutions (1) and (3) enabled us to estimate the correction necessitated by the thermal capacity of the heat source and to eliminate the error resulting from the use of the approximate expression (2).

Using the Minsk-22 electronic computer, we obtained values for the integral in (3) over a wide range of values of the parameter Fo for $\omega = 0.001-2.5$, which covers practically any medium.

The results of the calculations, shown in the form of differences between (2) and (3) as functions of Fo for various values of ω , make it possible in each individual case to decide whether the method should be used for investigating heat conduction in specific substances.

We propose a simple method for estimating the correction to the solution (2). We show that the value of the correction must be determined not for one value of Fo, but for two values, since the working formula used is the relation (2).

The results obtained enable us to draw conclusions concerning the nonuniformity of the use of some universal value of Fo for which the solution (1) would correspond to the solution (3) within the limits of acceptable accuracy. The correction taking account of the thermal capacity of the heat source depends in each specific case on ω , the region, and the range of variation of Fo.

NOTATION

| | |
|--|---|
| λ, a | are the coefficients of thermal conductivity and thermal diffusivity of the medium under investigation; |
| c, ρ | are the thermal capacity and density of the medium; |
| c_0, ρ_0 | are the thermal capacity and density of the material of the heat source; |
| q | is the specific power of the linear heat source, W/m; |
| r_0 | is the radius of the source; |
| $T(r, \tau)$ | is the temperature at a distance r from the axis of the source at time τ counted from the instant at which the source begins to act; |
| $\gamma = 0.5772\dots$ | is the Euler's constant; |
| $\omega = 2 c\rho/c_0\rho_0; Fo = 4a\tau/r_0^2;$ | |
| x | is a parameter of integration; |
| $J_i(x), Y_i(x)$ | are the Bessel and Neumann functions of order i ; |
| $T^* = 4\pi\lambda T/q$ | is the reduced temperature. |

Dep. 2291-74, July 17, 1974.

Original article submitted September 26, 1973.

ESTIMATION OF LATERAL HEAT FLOW IN A NONUNIFORMLY HEATED PLATE

L. V. Zysin and Yu. A. Migrov

UDC 536.21

The two-dimensional steady-state thermal conduction is considered for plates consisting of one or two layers with a given arbitrary heat-flux distribution at one surface, together with boundary conditions of the third kind at the other and thermally insulated end surfaces.

Analytical solutions have been obtained for constant values of the heat-transfer coefficient and flow temperature. The numerical calculations were performed by computer for various heat-flux distributions on the heated surface. The large number of important parameters makes it difficult to present a general survey of the results and derive universal relationships, but in certain particular cases the problem can be simplified considerably.

The following parameter is used to estimate the heat flux along the plate:

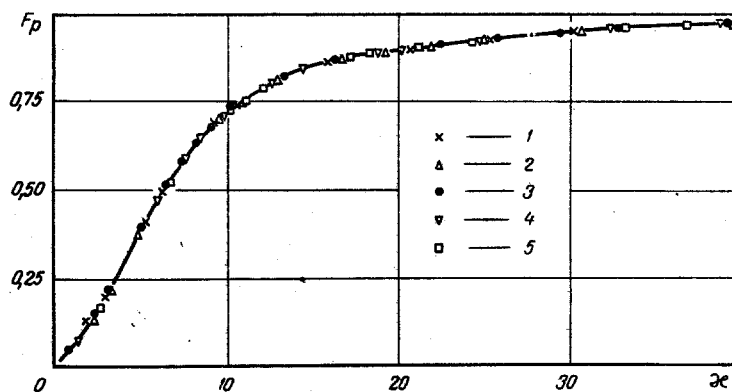


Fig. 1. Summary of calculated results for a one-layer plate with various heat-flux distributions at the input surface: 1) sinusoidal; 2) triangular; 3) linear sinusoidal; 4) linearly increasing; 5) sinusoidally increasing.

$$F_p = \frac{\int_0^a |\bar{q} - q_2(x)| dx}{\int_0^a |\bar{q} - q_1(x)| dx}$$

where \bar{q} is the mean integral heat flux, $q_1(x)$ is the heat flux at the input surface, x is the longitudinal coordinate on the plate, $q_2(x)$ is the heat flux at the output surface, and a is the dimensionless length of the plate.

If there is no heat flow along the plate, then $F_p \rightarrow 1$; the maximum flow corresponds to a uniform heat flux at the output surface ($F_p \rightarrow 0$).

It is shown that F_p is a single-valued function of the parameter $\kappa = a / \sqrt{(2Bi)/(2 + Bi)}$ for a one-layer plate in the realistic range of a (10-1000) and Bi (10^{-3} -10); a series of calculations confirms that the assumptions are justified. For heat-flux distributions that may be represented by smooth curves, the results agree to within $\pm 2\%$ with the general $F_p = F_p(\kappa)$ relationship that can be used directly in engineering calculations (Fig. 1).

Dep. 2292-74, July 8, 1974.

Original article submitted April 20, 1973.

REGULARIZATION OF THERMAL KINETICS FOR NONLINEAR BOUNDARY CONDITIONS

N. M. Tsirel'man

UDC 536.21

The temperature variation in an infinite plate heated by a constant-temperature radiator is determined by solving the appropriate boundary value problem by means of a computer. The calculation results, obtained for $\beta_0 = 0-0.80$ and $Sk = 0.01-5.0$, are interpreted as the shifting of $\Theta = \text{idem}$ isotherms in the solid (Fig. 1). The equidistance of segments of the different $\Theta = \text{idem}$ curves indicates that the isotherms shift at a constant local rate, which, as we proved earlier, is an indication of the regularization of thermal kinetics. Regular thermal conditions set in first in the central part of a solid and then extend to ever larger regions in time.

Analysis of the temperature changes for $\eta = \text{idem}$ in $-\ln(1 - \Theta)$ vs Fo and Θ vs Fo plots indicates that the steady-state thermal kinetics for boundary conditions involving the radiation law initially represents quasistationary thermal conditions, which then pass into regular thermal conditions of the first kind. The extent of the section of quasistationary conditions diminishes with an increase in Sk and β_0 ; however, the calculation relationships must account for the effect of each section of steady-state thermal kinetics and have the following form for the central part of the solid:

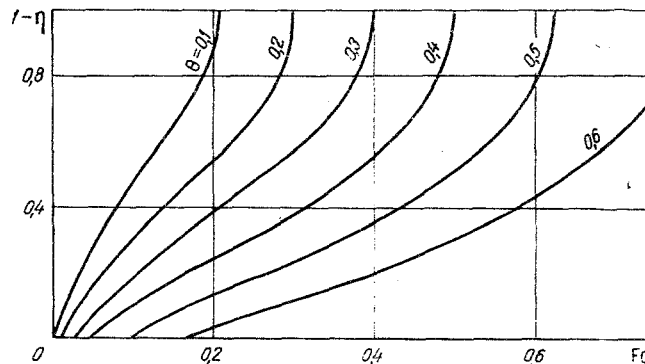


Fig. 1. Behavior of the isotherms $\Theta = \text{idem}$ for $\beta_0 = 0.5$ and $Sk = 1.0$.

$$\theta = \begin{cases} k \left(Fo - \frac{1}{6} + \frac{1}{2} \eta^2 \right) & Fo_{reg} \leq Fo \leq Fo_{II}, \\ 1 - A(p) \sqrt{1 - p\eta^2} \exp(-p Fo), & Fo_t \leq Fo \leq +\infty, \end{cases} \quad (1)$$

where Fo_t represents the time of transition from quasistationary conditions to regular conditions of the first kind. The value of k is determined from preliminary calculations. We have $k \cong Sk$ for $\vartheta_0 = 0$ if $Sk \leq 0.5$, while, for $Sk \geq 3.0$, the value of k is independent of ϑ_0 .

For small values of Sk , expression (1) describes the temperature field throughout the solid.

The first thermal amplitude $A(p)$ and p in (1) must be determined from the well-known solution of the third boundary-value problem for $Bi = 4 Sk$.

It is shown that the shift rate of isotherms in the central part of the plate is equal to

$$\bar{v}_\theta = \frac{a}{l_0} \left[\frac{\partial(l_0 - x)}{\partial \tau} \right]_\theta = \frac{l_0}{x} \quad (2)$$

and that expression (2) can be used for determining the thermal diffusivity a if the experimental behavior of the curves $T = \text{idem}$ ($\vartheta = \text{idem}$) in an $(l_0 - x)$ vs τ plot is first used to calculate $[\partial(l_0 - x)/\partial \tau]_\vartheta$ at the point $\eta = x/l_0$ that has become engaged in the steady-state thermal kinetics.

The conclusion is reached that regularization of thermal kinetics occurs for any type of monotonic heating of a solid for nonlinear boundary conditions.

NOTATION

| | |
|--|--|
| $T(x, \tau)$ and $\theta = [T(x, \tau) - T_0]/(T_R - T_0)$ | are the dimensional present temperature and dimensionless relative temperature, respectively; |
| T_0 and $\vartheta_0 = T_0/T_R$ | are the dimensional and dimensionless initial temperatures of the solid, respectively; |
| T_R | is the radiator temperature; |
| l_0, x , and $\eta = x/l_0$ | are the half-thickness of the plate, the dimensional coordinate, and the dimensionless coordinate, respectively; |
| τ and $Fo = a\tau/l_0^2$ | are the time and the Fourier number, respectively; |
| a | is the thermal diffusivity; |
| Sk | is the Stark number; |
| Bi | is the Biot number. |

Dep. 2279-74, April 17, 1974.

Original article submitted July 12, 1973.

THERMAL CONDUCTIVITY OF SOLIDS AT THE INITIAL STAGE OF HEATING

Yu. V. Vidin and Yu. A. Pshenichnov

UDC 536.21

The solution of the one-dimensional symmetric boundary-value problem of nonstationary thermal conductivity in solids of classical form ($\Gamma = 0, 1, 2$) with a nonuniform initial temperature distribution $f(x)$ for the ambient temperature $\theta_a(Fo)$ and the surface heat flux $Ki(Fo)$, which depend on the time, is given by [1]

$$\theta = A + \sum_{n=1}^{\infty} U_n(X) \exp -\mu_n^2 Fo \left[\int_0^1 X^{\Gamma} U_n f dX + U_n(1) \int_0^{Fo} \varphi(\eta) \exp \mu_n^2 \eta d\eta \right], \quad (1)$$

where A is the term of the series corresponding to the zero value of the root μ , and

$$U_n(X) = \sqrt{2C_n} V_{\Gamma}^{-1}(\mu_n) W_{\Gamma}(\mu_n X), \quad \varphi(Fo) = Bi \theta_a(Fo) + Ki(Fo).$$

However, the solution in this form is not very useful for determining the temperature for small values of Fo because of the slow convergence of the infinite series in this expression.

Using the properties of eigenfunctions, namely,

$$U_n = -\frac{1}{\mu_n^2} \left(\frac{d^2 U_n}{dX^2} + \frac{\Gamma}{X} \frac{dU_n}{dX} \right), \quad \frac{dU_n(0)}{dX} = 0, \quad \frac{dU_n(1)}{dX} + \text{Bi} U_n(1) = 0,$$

and the formula for integration by parts, we transform the expression for the temperature field (1) and obtain the following:

$$\theta = A + [\varphi(\text{Fo}) + A_1] P_\Gamma(X) + A_1 [A^* - \theta^*(X, \text{Fo})] - \sum_{n=1}^{\infty} \mu_n^{-2} U_n(X) \exp -\mu_n^2 \text{Fo} \times \\ \times \left[\int_0^1 X^\Gamma U_n \left(\frac{d^2 f}{dX^2} + \frac{\Gamma}{X} \cdot \frac{df}{dX} \right) dX + U_n(1) \int_0^{\text{Fo}} \varphi'(\eta) \exp \mu_n^2 \eta d\eta \right], \quad (2)$$

where

$$A_1 = \frac{df(1)}{dX} + \text{Bi} f(1) - \varphi(0), \quad P_\Gamma(X) = \sum_{n=1}^{\infty} \mu_n^{-2} U_n(1) U_n(X),$$

$$A^* = 0 \text{ for } \text{Bi} \neq 0, \quad A^* = (\Gamma + 1) \text{Fo} \text{ for } \text{Bi} = 0,$$

$$\theta^*(X, \text{Fo}) = A^* + P_\Gamma(X) - \sum_{n=1}^{\infty} \mu_n^{-2} U_n(1) U_n(X) \exp -\mu_n^2 \text{Fo}.$$

We find the sum of the series $P_\Gamma(X)$ by integrating the quasistationary problem corresponding to the initial one [1]:

$$P_\Gamma(X) = \frac{1}{\text{Bi}} \text{ for } \text{Bi} \neq 0, \quad P_\Gamma(X) = \frac{1}{2} \left(X^2 - \frac{\Gamma + 1}{\Gamma + 3} \right) \text{ for } \text{Bi} = 0.$$

The function $\theta^*(X, \text{Fo})$ is the solution of the problem stated for $f(X) = 0$ and $\varphi(\text{Fo}) = 1$.

Since $\theta^*(X, \text{Fo})$ has also been obtained in the Laplace form [1], while the series in (2) can be replaced by one or two terms because of the additional factor μ_n^{-2} , which improves convergence, the transformed solution (2) can be recommended for efficient calculation of the temperature at the initial stage of heating for arbitrary nonuniform boundary conditions.

Thus, the proposed method consists in separating the fast-converging series from the solution in traditional form and replacing the remaining part, which constitutes the solution of the simple problem, by its analog in the Laplace form, which holds and is convenient in the case of small Fourier numbers.

This method allows extension to two- and three-dimensional problems of nonstationary thermal conductivity in solids of classical form for both symmetric and nonsymmetric boundary conditions.

LITERATURE CITED

1. A. V. Lykov, Heat and Mass Exchange, Manual [in Russian], Energiya, Moscow (1972).
Dep. 2282-74, May 15, 1974.
Krasnoyarsk Polytechnic Institute.
Original article submitted May 18, 1973.

THE THERMAL PROCESSES IN GRINDING

V. A. Prilutskii and V. M. Khorol'skii

UDC 621.923.001.5:536.4

The steady-state heat transfer in grinding is considered for a system of three bodies: grinding wheel, component, and resulting powder. The following heat-balance equation is applied to the grinding zone:

$$Q = Q_1 + Q_2 + Q_3,$$

where Q_i , $i = 1, 2, 3$ are the heat fluxes to the wheel, components, and powder, with P_z the tangential

component of the cutting force, $V = V_w$ the speed of the wheel, and $Q = P_z V$ the total power of the thermal sources. Nonuniformity in the heat flux in the contact regions is neglected, i.e., averaged heat fluxes are envisaged. The maximum temperatures in the contact areas are put for each body in the form

$$T = Q_i \gamma_i,$$

where γ_i , $i = 1, 2, 3$ are the thermal potentials that satisfy the equation of thermal conductivity and are determined by the initial and boundary conditions.

The expressions for the heat fluxes in terms of the overall temperature T are substituted into the heat-balance equation to get a solution for the heat-transfer problem:

$$T = Q \left[\sum_{i=1}^3 \frac{1}{\gamma_i} \right]^{-1}; \quad Q_i = T \frac{1}{\gamma_i}; \quad i = 1, 2, 3.$$

The main difficulty in this scheme is to calculate the thermal potentials γ_i , which are dependent on the speeds of the heat sources, the geometry of the body, and the sizes of the contact areas, as well as other factors. The heat-source method is used to obtain expressions for the thermal potentials for various conditions. For instance, when the component is taken as a half-space, while the grinding wheel is massive, we obtain

$$\gamma_3 = F \cdot V_c \frac{\lambda_3}{\omega_3}; \quad \gamma_1 = \frac{1}{2b\lambda_1} F \left(\frac{V_w l}{4\omega_1} \right); \quad \gamma_2 = \frac{1}{2b\lambda_2} F \left(\frac{V_c l}{4\omega_2} \right),$$

where b and l are the width and length of the contact area, ν , λ , ω are the coefficients of heat transfer, thermal conduction, and thermal diffusivity, F is the cross-sectional area of the resulting product, and V_c is the speed of the component;

$$F(\xi) = \frac{1}{\pi \xi^2} \{ 2\xi \exp(2\xi) [K_0(2\xi) + K_1(2\xi)] - 1 \},$$

where $K_0(x)$, $K_1(x)$ is the Macdonald function.

For ξ large the following asymptotic formula applies:

$$F(\xi) = \sqrt{\frac{4}{\pi \xi}} \left(1 - \sum_{k=1}^{\infty} \frac{(-1)^k \{(2k-1)!!\}^2}{2^{2k} (2k-1) k!} \xi^{-k} \right).$$

The solution has been examined in relation to the speeds of wheel and component. In particular, agreement is obtained with the well-known Isaev-Silin formulas for high speeds.

A formula has been derived for the relative temperature in relation to the constant component t_0 and variable component $\varepsilon\varphi(\tau)$ for the grinding depth $t = t_0 + \varepsilon\varphi(\tau)$, which arises, in particular, on account of the roughness:

$$T_0 = \frac{T - T_c}{T_c} = e\sigma\varphi(\tau) + e^2 \frac{\sigma(\sigma-1)}{2!} \varphi^2(\tau),$$

where $\sigma = \alpha - 0.25$, α is the power of t in the empirical formula for the tangential grinding force, $e = \varepsilon/t_0$ is the relative amplitude, and T and T_c are temperatures due to the variable and constant components of the grinding depth.

Dep. 2293-74, April 16, 1974.

V. V. Kuibyshev Polytechnic Institute.

Original article submitted April 24, 1973.

THERMAL CONDUCTIVITY OF LIQUID n-HEPTANE

A. M. Mamedov

UDC 536.22

Available data on the thermal conductivity of liquid n-heptane agree within 5-8%, which hinders objective evaluation of the accuracy of any individual study. The present paper correlates the transport properties with the equation of state to establish the most likely of the values.

It has been found [1] that the equation of state and the transport properties can be described by relationships for the same form.

The p, v, T data [2] indicate that the equation of state for n-heptane from 0 to 300°C and p up to 3000 bars is

$$\frac{pv}{RT} = 1 + C\rho^2 + G\rho^6 \quad (1)$$

Then the thermal conductivity and dynamic viscosity of liquid n-heptane should be expressed by the following equations analogous to (1):

$$\lambda/\lambda'_s = 1 + C_\lambda \rho^2 + G_\lambda \rho^6 \quad (2)$$

$$\eta/\eta'_s = 1 + C_\eta \rho^2 + G_\eta \rho^6 \quad (3)$$

These equations have been confirmed from the linearity of the isotherms or combinations of the thermal conductivity and viscosity for n-heptane in the following coordinate systems:

$$\left(\frac{\lambda}{\lambda'_s} - 1\right)/\rho^2, \rho^4 \text{ and } \left(\frac{\eta}{\eta'_s} - 1\right)/\rho^2, \rho^4.$$

Further, the relationship between the thermal conductivity and viscosity will be linear at a given temperature. The equation for this linear relationship is derived as follows: with $\lambda = \lambda'_s$ and $\eta = \eta'_s$ we should have from (2) and (3) that

$$-C_\lambda/G_\lambda = -C_\eta/G_\eta = \rho_s^4 \quad (4)$$

Then (2) and (3) can be put in the form

$$\left(\frac{\lambda}{\lambda'_s} - 1\right)/G_\lambda \rho^2 = \rho^4 - \rho_s^4 \quad (2')$$

$$\left(\frac{\eta}{\eta'_s} - 1\right)/G_\eta \rho^2 = \rho^4 - \rho_s^4 \quad (3')$$

and these are equated with (4) to obtain

$$\lambda = m(t) + n(t)\eta \quad (5)$$

where

$$m(t) = \left(1 - \frac{C_\eta}{C_\lambda}\right) \lambda'_s; \quad n(t) = \frac{\lambda'_s}{\eta'_s} \cdot \frac{C_\eta}{C_\lambda}$$

Then this method allows one to establish objectively the accuracy of any particular set of data for λ and η and also indicates the conformity between the thermophysical parameters.

As (1) describes the p, v, T properties of n-heptane up to 300°C and 3000 bar, one might be able to describe the transport properties of this substance up to these levels; however, there are no experimental data on the viscosity and thermal conductivity in these ranges, so it is not presently possible to check these calculated values.

NOTATION

| | |
|---|--|
| P | is the absolute pressure, bars; |
| T | is the absolute temperature, °K; |
| v(p, T) | is the specific volume of liquid, cm ³ /g; |
| R = 0.82979 bar · cm ³ /g · deg | is the universal gas constant; |
| ρ(p, T) | is the liquid density, g/cm ³ ; |
| ρ _S (T) | is the same, for the saturated liquid, g/cm ³ ; |
| λ(p, T) | is the thermal conductivity W/m · deg; |
| λ' _S (T) | is the same for the saturated liquid; |
| η(p, T) | is the dynamic viscosity N · sec/m ² ; |
| η' _S (T) | is the same for the saturated liquid; |
| C, G, C _λ , G _λ , C _η , and G _η | are the coefficients in (1), (2), and (3) dependent only on T. |

LITERATURE CITED

1. A. M. Mamedov and T. S. Akhundov, *Inzh.-Fiz. Zh.*, **23**, No. 3 (1972).
2. K. A. Doolittle, "Specific volumes of n-alkanes," *J. of Chemical Engineering, Data*, **9**, No. 2 (1964).

Dep. 2289-74, May 15, 1974.

Original article submitted January 29, 1973.

CRITICAL HEATING RATE IN A HEAT EXCHANGER WITH MAGNETIC WATER TREATMENT

M. L. Mikhel'son and L. E. Abramov

UDC 541.123:538.6:621.187.3

Magnetic treatment in a heat exchanger acts by reducing the saturation concentration required to initiate crystallization, with centers activated by the magnetic field.

To ensure that the crystallization occurs within the water instead of at the surfaces of the tube, one has to meet the condition $\rho_{sm} < \rho < \rho_s$, where $\rho_{sm} = \rho_s \exp(-u/kT)$; u is the activation energy, and this can be found by experiment.

This condition imposes a restriction on the heating rate \dot{T} , which is limited by the diffusion rate, concentration N , and particle radius r .

The following system of equations is solved approximately:

$$\begin{aligned} r\dot{r} &= Dq/\gamma; \quad q = \rho - \rho_{sm} \\ \dot{q} &= -4\pi Dq \int_0^\infty r f(r, t) dr - \dot{\rho}_{sm} \\ \frac{\partial f}{\partial t} + \frac{\partial(rf)}{\partial r} &= 0 \end{aligned} \quad (1)$$

subject to the conditions $r_0^2/2(r^2 - r_0^2) \ll 1$ and $u/kT \ll 1$.

A turning point is identified for the concentration of the substance that crystallizes from the solution for $dq/dT = 0$; a formula is derived for the critical heating rate \dot{T}_{cr} , below which the deposition of scale is largely prevented:

$$\dot{T} < \dot{T}_{cr} = \frac{2D(6\pi^2 N^2/\gamma)^{\frac{1}{3}} \rho_s u [\rho(0) - \rho_s(1 - u/kT)]^{\frac{1}{3}}}{kT \left(\frac{d\rho_s}{dT} - \frac{\rho_s u}{kT^2} \right)} \quad (2)$$

Theoretical calculation has been performed for the heating rate for raw water in a fast water heater. The calculated critical rate is close to that used in practice.

NOTATION

- q is the supersaturation;
- ρ_{sm} is the equilibrium solute concentration near surface of particle activated by the magnetic field;
- ρ is the solute concentration in the water;
- D is the diffusion coefficient;
- γ is the particle density;
- f is the particle-radius distribution.

Dep. 2300-74, July 28, 1974.

Belinskii State Pedagogic Institute, Penza.

Original article submitted July 5, 1973.

MEASUREMENT OF THE ACTIVATION ENERGY
OF CRYSTALLIZATION CENTERS IN A
MAGNETIC FIELD

M. L. Mikhel'son

UDC 541.123:538.6

If a magnetic field acts for a short time on an aqueous solution containing suspended particles, the crystallization of the hardness salts will be effective; more crystallization centers are involved, and the crystal size spread is altered.

The effect of the field on the particle activity is represented by the exponential expression

$$\rho_{sm} = \rho_s \exp\left(-\frac{u}{kT}\right). \quad (1)$$

The paper shows how measurements can be used to find the activation energy u , which enables one to calculate the crystallization rate for magnetically treated solutions.

The method consists in counting the crystallization centers for water treated in the static state (in the thermostatic cell) and in a flow, in both cases using a VDK-4 ultramicroscope. The chemical composition of the aqueous solution has also been calculated for a variety of temperatures and the corresponding equilibrium concentration of the scale material.

The necessary counting time has been calculated on the basis of the crystallization and sedimentation kinetics.

Experiment shows that the logarithm of the particle concentration increases linearly with the water temperature. The straight line for the magnetized case lies higher. The activation energy can be calculated from the formula and expressed in terms of the equilibrium concentrations of calcium carbonate in solutions corresponding to different temperatures T_1 and T_2 , but with identical particle concentrations for the magnetized and unmagnetized cases.

It is found that the particle activation is dependent on the magnetic field strength in an oscillatory fashion; the maximum values of the activation energy are about $3 \cdot 10^{-15}$ erg in the range 20-70°C, or about 7.2% of the thermal energy kT .

NOTATION

ρ_{sm} is the equilibrium solute concentration near surface of particle activated by the magnetic field;
 ρ_s is the same, in the absence of the field;
 k is the Boltzmann's constant.

Dep. 2299-74, July 28, 1974.

Original article submitted July 5, 1973.

TEMPERATURE DISTRIBUTION PRODUCED BY
A PERIODICALLY VARYING RING SOURCE IN A
TWO-LAYER MEDIUM WITH A CYLINDRICAL INTERFACE

V. K. Alekseev

UDC 536.212

The steady-state purely periodic state is considered, with the temperature distribution in the form $T_1 = (u_0 + u_1)e^{-i\omega t}$ for the region $a \leq r < \infty$ with λ_1, c_1 ; $T_2 = u_2e^{-i\omega t}$ and for the region $0 \leq r < a$ for λ_2 and c_2 . The function $u_0(r, z)$ corresponds to an elementary annular source of radius $r_0 > a$ with a periodically varying output, while $u_1(r; z)$ and $u_2(r; z)$ may be represented as Fourier integrals whose spectral

densities contain the functional multipliers $q_1(r)$ and $q_2(r)$; the conditions for field linkup at the interface between the media $r = a$ enable one to determine these factors. The integral expressions can be used as initial ones in numerical calculations on temperature distributions.

In the case of the external space $a \leq r < \infty$, the saddle-point method can be used to calculate the improper integral with infinite limits, which gives an analytical expression for numerical use in the remote zone in the spherical coordinates R and θ .

NOTATION

| | |
|-----------------------|----------------------------------|
| $r, \varphi,$ and z | are the cylindrical coordinates; |
| a | is the radius of interface; |
| λ | is the thermal conductivity; |
| c | is the thermal diffusivity; |
| ω | is the frequency; |
| t | is the time; |
| T | is the temperature; |
| r_0 | is the source radius; |
| R and θ | are the spherical coordinates. |

Dep. 2703-74, April 18, 1974.

Original article submitted January 1, 1973.

GRAPHICAL METHOD OF DETERMINING THE ATTENUATION OF TEMPERATURE FLUCTUATIONS IN THE OUTER WALLS OF BUILDINGS

V. V. Nasedkin and A. P. Nasedkina

UDC 536.21

Temperature-oscillation damping is considered for a wall consisting of 1, 2, or 3 layers; generalized variables have been derived on the basis of the heat-uptake coefficient ξ [1], which is the heat-transfer parameter for harmonic temperature oscillation. The temperature amplitude is represented as a function of the following quantities:

$$\delta \sqrt{\frac{\omega}{a}} = \frac{\delta}{\xi} = D, \quad (1)$$

$$x \sqrt{\frac{\omega}{a}} = \frac{x}{\xi} = D_x, \quad (2)$$

$$\lambda \sqrt{\frac{\omega}{a}} = \frac{\lambda}{\xi} = S, \quad (3)$$

with D and D_x the relative thickness of a layer and a coordinate whose unit of measurement is ξ . From (3) we see that S is the thermal conductivity of a layer whose thickness is ξ or a relative dimensionless unity. Then S may be defined as the thermal conductivity coefficient [2].

If the layer thicknesses are represented as the relative quantities D , then the boundary condition of the fourth kind is represented by

$$\frac{\operatorname{tg} \psi_{n-1}}{\operatorname{tg} \psi_n} = \frac{S_n}{S_{n-1}}. \quad (4)$$

The oscillation amplitudes in a layer have been examined in relation to the generalized variables D and D_x together with the ratio S_n/S_{n-1} ; it is concluded that the damping within the thickness of a layer can be represented roughly as follows: on the inner surface of the layer in a part whose relative thickness is unity the amplitude change is linear, and then, out to the outer surface, it is as in a semi-infinite body, i. e., varies exponentially. If the relative layer thickness is less than unity, then the amplitude variation throughout the thickness may be approximated by a straight line.

An approximate method is given for determining the temperature oscillation in a multilayer wall. The method is illustrated by reference to a three-layer wall.

LITERATURE CITED

1. A. V. Lykov, Theory of Thermal Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
2. V. V. Nasedkin, "Propagation of heat waves in a semi-infinite body," in: Aspects of Heating and Ventilation [in Russian], RISI, Rostov-on-Don (1973).

Dep. 2316-74, March 20, 1974.

Original article submitted March 12, 1973.

DISTRIBUTION OF THE SOLID DISPERSE PHASE OF AN AEROSOL DURING THE DIRECTIONAL FILTRATION OF A DUSTY GAS

V. P. Kurkin

UDC 697.942

The distribution of the solid disperse phase of an aerosol on the surface of the porous element during the filtration of a dusty gas gives rise to the operating properties of the majority of the gas-cleaning devices which operate by the filtration method.

Theoretical studies have shown that the process of directional deposit of the solid disperse phase of an aerosol during filtration is described by a nonlinear differential equation in partial derivatives. For a particular case the problem is reduced to the solution of a linear second-order equation in partial derivatives

$$\frac{\partial^2 \eta}{\partial x^2} = a^2 \frac{\partial \eta}{\partial t} \quad (1)$$

with the following boundary and initial conditions:

$$\eta(0; t) = \eta_0; \quad \eta(x; 0) = 0; \quad \lim_{x \rightarrow L} \eta(x; t) = 1.$$

The solution of Eq. (1), keeping in mind that $x \leq L$, is written in the form

$$\eta = 1 - \operatorname{erf} \left[\frac{a}{2} \cdot \frac{L-x}{\sqrt{t}} \right], \quad (2)$$

where $a = (\tau/k)^{0.5}$.

When considering a problem with axial symmetry, as in filtration through bags, the solution of (2) will have the form

$$\eta = \frac{1}{2} \left\{ 1 + \operatorname{erf} \left[-\frac{a}{2} \frac{L-x}{\sqrt{t}} \right] \right\}. \quad (3)$$

In the normal (Gaussian) coordinate system this equation represents the equation of a straight line intersecting the abscissa which passes through the middle of the ordinate scale at the point $x = L$. The tangent of the angle of inclination of this straight line to the abscissa on which the value $L - x$ is plotted is equal to $-a/2\sqrt{t}$.

The distribution of dust along the length of the bag, which characterizes the mass dust content in each cross section of the bag, can be determined through the differentiation of the integral function (3). The height of the maximum in the differential function is determined by the mean square deviation of the integral distribution and is numerically equal to the tangent of the angle of inclination of the corresponding straight line.

Experimental studies were made of the process of filtration through lengthwise filter bags (of height 8) which agree with the theoretical results.

The studies conducted made it possible to clarify one of the most important properties of the process of covering a filter bag with dust, responsible for the feasibility of the engineering calculation of this process.

NOTATION

| | |
|--------------------------|---|
| $\eta = (G_0 - G_i)/G_0$ | is the relative dustiness; |
| G_0, G_i | are the mass of solid disperse phase in gas at entrance and in i -th cross section, kg/sec; |
| x | is the current coordinate, m; |
| L | is the length of bag, m; |
| t | is the time, sec; |
| $\tau = 2r^2 \rho/\nu$ | is the relaxation time of aerosol particles, sec; |
| r | is the particle radius, m; |
| ρ | is the density, kg/m ³ ; |
| ν | is the viscosity, kg/m·sec; |
| $\operatorname{erf} \xi$ | is the error function; |
| k | is the permeability, m ² . |

Dep. 2270-74, May 12, 1973.

Original article submitted June 10, 1972.

THEORY OF EVAPORATIVE COOLING OF WATER

V. P. Alekseev and A. V. Doroshenko

UDC 621.175.3:621.565.93

The model of an "ideal" countercurrent cooling tower [1], which is characterized by the conditions of thermodynamic equilibrium at the lower ($t_W^2 = \tau_G^1$) and upper ($i_G^2 = i_{GW}^1$) ends of the apparatus, is used in an analysis of the processes of evaporative cooling in a cooling tower. A graphic interpretation of the model on an i vs t diagram is proposed, and a nomogram is calculated which allows one to find the values of the relative minimum flow rate l_{id} of air as a function of the water temperature t_W^1 at the entrance to the apparatus and the wet-bulb thermometer temperature τ_G^1 of the entering air. Along with the degree of cooling $E_W = (t_W^1 - t_W^2)/(t_W^1 - \tau_G^1)$ of the water the degree of utilization $E_G = (i_G^2 - i_G^1)/(i_{GW}^1 - i_G^1)$ of the air is analyzed, and the limiting values of these quantities as functions of the characteristic number Λ are established ($\Lambda = l/l_{id}$, where $l = G/W$ is the relative flow rate of air).

It is shown that in the general case the limits of occurrence of the process in a cooling tower are determined by the value of l , in addition to the values of t_W^1 and τ_G^1 . In connection with this a refined model of an "ideal" cooling tower is developed, and the concept of the degree of efficiency E of the process, the value of which characterizes the process as a whole both from the aspect of the efficiency of water cooling and of air utilization, is introduced. The limiting values of E are established: $E_{lim} \approx \Lambda$ when $\Lambda \leq 1$; $E_{lim} \approx 1/\Lambda$ when $\Lambda \geq 1$.

An experimental study is performed on models of a film cooling tower, where adapters of regular structure made of thin-walled sheets of aluminum and miplast [2] are used as the exchange surface. It is established that the dependence $E = f(\Lambda)$ has an extremum maximum (E_*) at the point with $\Lambda = 1$. Replotting of the experimental data in the reduced coordinates $\bar{E} = E/E_*$ led to the equation

$$\bar{E} = 2.25 (1 - e^{-1.1\Lambda})^2 \Lambda^{-1},$$

which describes the reduced characteristic \bar{E} of the cooling tower irrespective of the type of apparatus, the structural properties of the adapter, the material, or the operating conditions. It is shown that the results obtained can be used in equal measure in the analysis of processes in a cross-current cooling tower and in the heat-exchange devices of air-conditioning systems.

LITERATURE CITED

1. W. Klenke, Brennst.-Wärme-Kraft, 18, 97 (1966); Kältetechnik-Klimatisierung, 22, 322 (1970).
2. V. P. Alekseev, A. V. Doroshenko, and N. G. Vasil'eva, Izv. Vuz. SSSR, Energetika, No. 12 (1971); Izv. Vuz. SSSR, Khim. i Khim. Tekh., 16, No. 3 (1973).
3. V. P. Alekseev (Aleksjejev), A. V. Doroshenko (Doroszenko), and N. G. Vasil'eva (Vasiljeva), Sb. Konferencja Naukowo-Techniczna nt Wymienniki Ciepła Urzadzen Chlodniczych i Klimatyzacyjnych. Rzeszow (Polska), 18-19 Wrzesien, 1973.

Dep. 2307-74, May 15, 1974.

Original article submitted February 6, 1973.

TEMPERATURE DISTRIBUTION IN A HOLLOW CYLINDER WHEN HEATED ALONG A SPIRAL

V. N. Maksimovich and G. V. Plyatsko

UDC 536.24;511.15

Equations are presented for determining the temperature field in a hollow cylinder when its surfaces are heated along a spiral.

The temperature field in a hollow cylinder is determined for the case when the heat exchange with both its surfaces and the initial conditions can be represented in the form

$$\left[\frac{\partial T}{\partial r} + (-1)^j \alpha_j \lambda^{-1} T \right]_{r=r_j} = \alpha_j \lambda^{-1} \varphi_j(t_j, \tau), \quad j = 1, 2, \quad (1)$$

$$T|_{\tau=0} = \sum_{i=1}^2 f_i(r, t_i), \quad t_i = k_1^i \beta_1^i + k_2^i z \cdot r_i^{-1}, \quad \beta_1^i = 2\pi [n_i] + \beta,$$

$[n_i]$ is the integral part of the number of turns of one of the spiral lines at the surface r_i inclined at the angle $(\pi/2) - (\gamma_i)$ to the generatrix from the cross section $z = 0$ to the point (r_i, β, z) , where $|t_i| \leq l_i = \pi R_i^1$, $i = 1, 2$; $\varphi_i(t_i)$ is the temperature of the surrounding medium.

The solution of the heat-conduction equation with the boundary conditions (1) is represented in the form

$$T = T_1 + T_2,$$

where T_1 and T_2 satisfy the heat conduction equations and the conditions

$$\left[\frac{\partial T_1}{\partial r} + \alpha_2' T_1 \right]_{r=r_2} = \alpha_2' \varphi_2(t_2), \quad \left[\frac{\partial T_1}{\partial r} - \alpha_1' T_1 \right]_{r=r_1} = 0,$$

$$\left[\frac{\partial T_2}{\partial r} + \alpha_2' T_2 \right]_{r=r_2} = 0, \quad \left[\frac{\partial T_2}{\partial r} - \alpha_1' T_2 \right]_{r=r_1} = \alpha_1' \varphi_1(t_1),$$

$$T_1|_{\tau=0} = f_2(r, t_2), \quad T_2|_{\tau=0} = f_1(r, t_1), \quad \alpha_j' = \frac{\alpha_j}{\lambda}.$$

Thus, the problem is reduced to the solution of the heat-conduction equation with the boundary conditions ($t = t_1, t_2$)

$$\left[\frac{\partial T}{\partial r} + (-1)^j \alpha_j' T \right]_{r=r_j} = \psi_j(t, \tau), \quad j = 1, 2, \quad T|_{\tau=0} = f_0(r, t). \quad (2)$$

In the case of the stationary problem with the conditions (2) and

$$\psi_j(t, \tau) = \sum_{n=0}^{\infty} a_n^j \cos \frac{n\pi}{l} t, \quad j = 1, 2$$

the solution is written as

$$T = \sum_{n=0}^{\infty} T_n(r) \cos \frac{n\pi}{l} t,$$

where

$$T_n(r) = A_n I_n(\alpha_n) + B_n K_n(\alpha_n), \quad \alpha_n = kn \frac{r}{r_1}.$$

The constants A_n and B_n are determined from the system of equations ($j = 1, 2$)

$$\left\{ A_n \left[\frac{kn}{r_1} I_n'(\alpha_n) + (-1)^j \alpha_j I_n(\alpha_n) \right] + B_n \left[\frac{kn}{r_1} K_n'(\alpha_n) + (-1)^j K_n(\alpha_n) \right] \right\}_{r=r_j} = a_n^j.$$

The nonstationary problem, when the temperature of the surrounding medium varies according to a power law, is also examined.

NOTATION

| | |
|---------------------------|---|
| r | is the radius of cylinder; |
| r_1 and r_2 | are the inner and outer radii of cylinder; |
| β | is the angular coordinate; |
| z | is the axial coordinate; |
| T | is the temperature; |
| λ | is the thermal conductivity coefficient; |
| α_j | is the heat-release coefficient at outer and inner surfaces; |
| $I_\nu(x)$ and $K_\nu(x)$ | are the Bessel functions of imaginary argument; |
| τ | is the time; |
| γ_i | is the angle characterizing the inclination of the coil to the cylinder axis at the inner ($i = 1$) and outer ($i = 2$) surfaces, $k_1^i = \cos \gamma_i$, $k_2^i = \sin \gamma_i$. |

Dep. 2294-74, May 27, 1974.

Original article submitted July 3, 1972.

CALCULATION OF TEMPERATURE FIELDS IN THERMALLY INSULATED RODS AND SPHERES WITH ALLOWANCE FOR THE FINITE RATE OF HEAT SPREAD

M. A. Benyakovskii

UDC 536.21

In the present report the temperature field is calculated on the basis of a hyperbolic equation of heat conduction. In the case of a rod it is assumed that a temperature peak $T(x, 0) = b\delta(x)$ is created in the middle of it at the starting time. Using a Laplace transformation and a finite Fourier cosine transformation one obtains a general solution

$$\theta = 1 + 2 \exp\left(-\frac{1}{2} z^2 Fo\right) \sum_{n=1}^{\infty} \left[\operatorname{ch} \frac{1}{2} z^2 Fo \sqrt{1 - \left(\frac{n\pi}{z}\right)^2} + \frac{\operatorname{sh} \frac{1}{2} z^2 Fo \sqrt{1 - \left(\frac{n\pi}{z}\right)^2}}{\sqrt{1 - \left(\frac{n\pi}{z}\right)^2}} \right] \cos n\pi \frac{x}{l} \quad (1)$$

and a solution for short times

$$\theta = \frac{1}{2} z \exp\left(-\frac{1}{2} z^2 Fo\right) \sum_{k=1}^{k=5} \left[I_0\left(\frac{1}{2} z^2 Fo \sqrt{1 - \left[\frac{L_k}{lFo z}\right]^2}\right) + \frac{I_1\left(\frac{1}{2} z^2 Fo \sqrt{1 - \left[\frac{L_k}{lFo z}\right]^2}\right)}{\sqrt{1 - \left[\frac{L_k}{lFo z}\right]^2}} \right] u\left[ct\left(1 - \frac{L_k}{lFo z}\right)\right]. \quad (2)$$

Here $\theta = T(x, t)/T_0$ is the relative temperature; T_0 is the equilibrium temperature; $2l$ is the length of the rod; a is the thermal diffusivity; c is the rate of heat spread; $z = cl/a$ is a dimensionless parameter; Fo is the Fourier number; I_0 and I_1 are modified Bessel functions of zeroth and first order, respectively; u is a unit step function; $L_1 = |x|$; $L_2 = 2l - x$; $L_3 = 2l + x$; $L_4 = 4l - x$; $L_5 = 4l + x$ [we neglect the remaining terms of the series of (2) because of their smallness].

By setting $k = 1$ we obtain the solution of the hyperbolic equation for an infinite region in the one-dimensional case. The subsequent terms ($k = 2, 3, 4 \dots$) describe the temperature waves reflected from the ends of the rod.

The wave nature of the process of spread of a heat pulse is manifested noticeably only for small z ($z < 10$). Assuming that $z \rightarrow \infty$ in Eqs. (1) and (2), we find the solutions of the classical heat-conduction equation for the given problem.

For a sphere the general solution has the following form:

$$\theta = 1 + \frac{2}{3} \frac{R}{r} \exp\left(-\frac{1}{2} z^2 \text{Fo}\right) \sum_{n=1}^{\infty} \left[\text{ch} \frac{1}{2} z^2 \text{Fo} \sqrt{1 - \left(\frac{2 \mu_n}{z}\right)^2} + \frac{\text{sh} \frac{1}{2} z^2 \text{Fo} \sqrt{1 - \left(\frac{2 \mu_n}{z}\right)^2}}{\sqrt{1 - \left(\frac{2 \mu_n}{z}\right)^2}} \right] \frac{\sin \frac{\mu_n}{R} r}{\sin \mu_n \cos \mu_n} \quad (3)$$

Here μ_n are the roots of the characteristic equation $\mu = \tan \mu$.

Dep. 2298-74, July 18, 1974.

Original article submitted May 16, 1973.